Core Properties from Superconducting Gravimeter Data

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Abstract

The emerging network of superconducting gravimeter installations, many supported by Japanese programmes, are giving an unusual source of information on properties of Earth's deep interior. The combination of the spectra from many different observatories into the Product Spectrum has led to the detection of the three translational modes of oscillation of Earth's solid inner core. We show how the reduction in the rotational splitting of the two equatorial modes, compared to the inviscid case, can lead to a measure of viscosity in the F-layer surrounding the inner core, and that inner core density is highly resolved by the axial mode period, which is not much split by rotation or viscosity.

1 Introduction

The analysis of superconducting gravimeter records has led to the detection of the three translational modes of oscillation of Earth's solid inner core both in European observations (Smylie et al, 1993), and in observations outside Europe (Courtier et al, 2000). Due to Earth's rotation, the modes are split with periods found at 3.5822 ± 0.0012, 3.7656 ± 0.0015, and 4.0150 ± 0.0010 hours. In this paper, we show how these observations can be used to determine the viscosity in the F-layer just outside the inner core and to confirm the inner core density of Earth model Cal8 of Bolt and Uhrhammer (Bullen and Bolt, 1985, Appendix) to within $2.2 \times 10^{-3}\ gm \cdot cm^{-3}$.

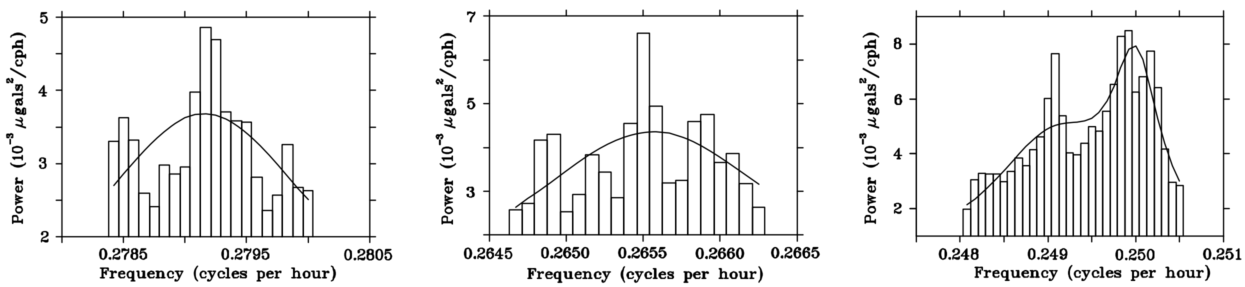

Figure 1: Product Spectra of (from left to right) the retrograde, axial and prograde modes from observations at Bad Homburg, Brussels, Cantley and Strasbourg (Courtier et al, 2000). The prograde mode is near the large solar heating tide feature S_6 at exactly six cycles per solar day. A full statistical analysis of the Product Spectrum and a method of calculating confidence intervals for it have been given previously (Smylie et al, 1993).

2 Observation and Interpretation of Inner Core Translational Modes

The resonances associated with the translational modes were first identified visually in a product of four individual spectra from long superconducting gravimeter records taken in Europe amounting to a total of $111,000$ hourly observations (Smylie et al, 1992). The Product Spectrum is a very useful technique where non-simultaneous observations at a number of stations are to be combined to look for common features and to minimize local station systematic errors. A full statistical analysis of the Product Spectrum, together with a method of calculating confidence intervals for it, is given by Smylie et al (Smylie et al, 1993). Strict adherence to a splitting law derived from elementary dynamics assists identification of the triplet of resonances. Largely independent of Earth model, it can be shown that the central frequencies must fall on splitting curves. The observed periods obey the splitting law to about four significant figures and a computer based search for equally significant triplets of resonances failed to find others (Smylie et al, 1993).

Now almost $300,000$ hours of superconducting gravimeter records have been analyzed, from both inside and outside Europe. In Figure 1 resonances from a Product Spectrum including observations at the Cantley, Québec station in Canada as well as those from Europe are shown.

The calculation of the viscous and pressure drags requires a model for the inviscid flow immediately outside the boundary layer surrounding the inner core. Since the effects of stratification and compressibility there are likely to be negligible, this flow field is given by solutions of the Poincaré equation. For a reference Earth frame rotating at mean rate Ω about a fixed spatial direction the angular frequency of the motion, ω, can be expressed by the dimensionless Coriolis frequency $\sigma = \omega/2\Omega$. Following Bryan (Bryan, 1889), the axis aligned with the rotation vector is stretched by the factor $1/\tau$ where $\tau^2 = 1 - 1/\sigma^2$. In the stretched 'auxillary' coordinates, the Poincaré equation becomes Laplace's equation.

Novel solutions of the Poincaré equation using the Legendre function of the second kind have been obtained (Smylie and McMillan, 1998), which allow the condition of continuity of the radial displacement to be satisfied exactly at the inner core boundary and asymptotically at the core-mantle boundary. Together with conservation of linear momentum between the inner core, outer core and shell, three equations are obtained whose solution in powers of $1/\sigma$ allow the pressure drag to be expressed in the form

$$4\Omega^2\left(\alpha\sigma^2+\beta\sigma+\gamma\right)U_I \tag{1}$$

with U_I representing the inner core displacement (Smylie and McMillan, 2000). It is found that

$$\alpha = M_I'\left(\frac{1}{2}+\frac{3}{2}\frac{M_I+(a/b)^3 M_S}{M_O+M_S\left(1-(a/b)^3\right)}\right), \tag{2}$$

for both the axial and equatorial modes, $\beta=0$ for the axial mode and

$$\beta = M_I'\left(\frac{1}{4}-\frac{3}{4}\frac{M_I+(a/b)^3 M_S}{M_O+M_S\left(1-(a/b)^3\right)}\right) \tag{3}$$

for the equatorial modes. Similar, but more complicated expressions for γ can be found but they are not used and are omitted. a is the radius of the inner core, b is the radius of the core-mantle boundary, M_I is the mass of the inner core, M_O is the mass of the outer core, M_S is the mass of the shell and $M_I' = 4/3\pi a^3\rho_0$ is the displaced mass with ρ_0 the density just outside the inner core. The displacements arising from solutions involving only Legendre functions of the first kind, widely used in rotating fluid dynamics (Greenspan, 1969), are pure translations in this case and play an essential role in the conservation of linear momentum.

The leading order boundary layer equations (Moore, 1978) for the extra displacement components required to adjust the exterior flow to the no-slip condition at the inner core boundary can be solved and matched to the exterior flow (Smylie and McMillan, 1998). In these equations the local and Coriolis accelerations are balanced by viscous forces in the standard Ekman layer approximation. The ratio of the latter to the former is expressed by the dimensionless Ekman number

$$E_k = \frac{\eta}{\Omega a^2\rho_0} \tag{4}$$

where η is the dynamic viscosity. Direct integration of the viscous stress acting on the inner core boundary yields the viscous drag forces.

The viscous drags can be written in terms of the corresponding pressure drags given by expressions (1) (2) and (3). For the axial mode with pressure drag D_p^a, the viscous drag is

$$D_v^a = \frac{1-i}{4}\sqrt{E_k}\left(D_p^a+4\Omega^2 M_I'\sigma^2 U_I\right) f^a(\sigma), \tag{5}$$

where

$$\begin{aligned} f^a(\sigma) = \Big\{ & 8\left[(\sigma+1)^{3/2}+(\sigma-1)^{3/2}\right] \\ & -\frac{16}{5}\left[(\sigma+1)^{5/2}-(\sigma-1)^{5/2}\right]\Big\}. \end{aligned} \tag{6}$$

For the equatorial modes with pressure drag D_p^e, the viscous drag is

$$D_v^e = \frac{1\mp i}{8}\sqrt{E_k}\left(D_p^e+4\Omega^2 M_I'\sigma(\sigma-1)U_I\right) f^e(\sigma), \tag{7}$$

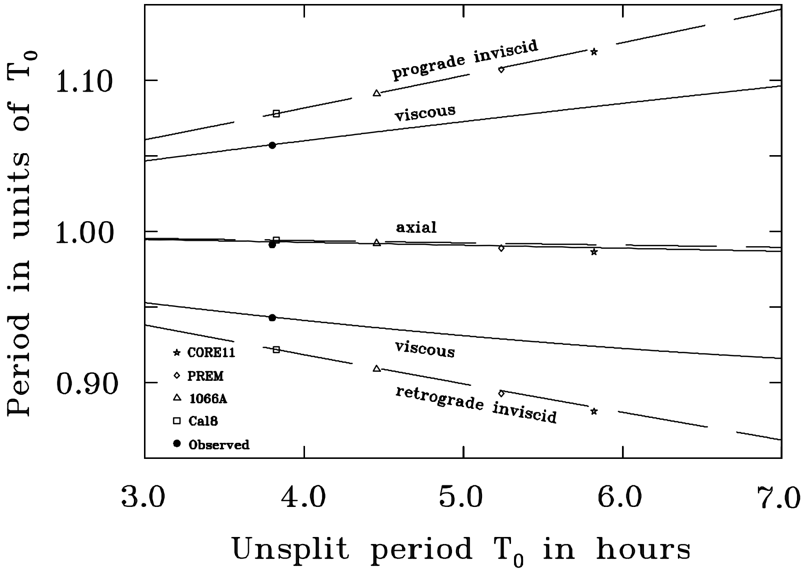

Figure 2: Splitting curves for the three translational modes. The inviscid curves for the three modes are shown dashed based on Earth model Cal8 (open squares). Inviscid periods are overplotted for Earth models Core11 (open stars) (Widmer et al, 1988), PREM (open diamonds) (Dziewonski and Anderson, 1981) and 1066A (open triangles) (Gilbert and Dziewonski, 1975). Solid viscous splitting curves are for a single viscosity of 1.243×10^{11} $Pa{\cdot}s$.

where $f^e(\sigma)$ is given by

$$f^e(\sigma) = \Big\{\mp 24\,(\pm\sigma\mp 1)^{1/2} - 16\,(\pm\sigma\mp 1)^{3/2} \\ -\frac{16}{5}\left[(\pm\sigma-1)^{5/2} - (\pm\sigma+1)^{5/2}\right]\Big\}. \tag{8}$$

The pressure drag form (1), together with the viscous drags given by expressions (5) and (7), allow us to write complex equations of motion for the axial and equatorial translational oscillations. In general, the real parts of these equations can be expressed as splitting laws for the period in the form

$$\left(\frac{T}{T_0}\right)^2 + 2g^v\frac{T_0}{T_S}\left(\frac{T}{T_0}\right) - 1 = 0, \tag{9}$$

where T is the period, T_0 is the unsplit period, T_S is the length of the sidereal day and g^v is a dimensionless viscous splitting parameter. In this form, if we plot T as a function of T/T_0, only g^v remains as a viscosity-dependent free parameter.

Plots of the splitting laws of the form (9) for all three modes are shown in Figure 2 for the parameters of Earth model Cal8 of Bolt and Uhrhammer (Bullen and Bolt, 1985, Appendix).

As illustrated in Figure 2, we have used the parameters of Earth model Cal8 to recover the viscosity although the result is not strongly dependent on Earth model.

Recovered viscosities and periods are listed in Table 1 for both Cal8 and CORE11 Earth models.

The density profiles for the four Earth models considered in this study are shown in Figure 3.

Table 1: Recovered viscosities and viscous periods.

Periods	Retrograde (hours)	Axial (hours)	Prograde (hours)
Observed Periods	3.5822 ± 0.0012	3.7656 ± 0.0015	4.0150 ± 0.0010
CORE11 Viscous Periods ($\nu = 9.58 \times 10^6\ m^2\ s^{-1}$, $\eta = 1.17 \times 10^{11}\ Pa \cdot s$)	3.5793	3.7647	4.0121
Cal8 Viscous Periods ($\nu = 10.21 \times 10^6\ m^2\ s^{-1}$, $\eta = 1.243 \times 10^{11}\ Pa \cdot s$)	3.5840	3.7731	4.0168
Cal8 Inviscid Periods	3.5168	3.7926	4.1118

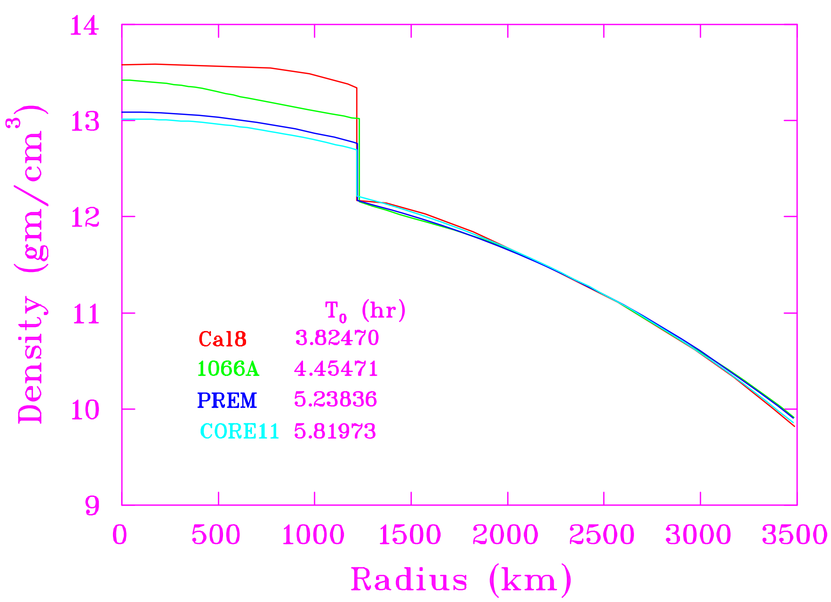

Figure 3: Density profiles in the inner and outer cores for Earth models Cal8, 1066A, PREM and Core11. The 0.6 $gm \cdot cm^{-3}$ density range in the inner core causes nearly a 2 h difference in the unsplit period T_0.

3 Discussion

The axial translational mode period is not much affected by rotation or viscosity while Coriolis splitting reduces the period of the retrograde equatorial mode and increases the period of the prograde equatorial mode, though not by as much as if the surrounding outer core fluid were inviscid. The reduction of the splitting of the two equatorial mode periods allows the inner core itself to be used as a kind of two-dimensional dynamic viscometer and a viscosity of $1.24 \times 10^{11}\ Pa \cdot s$ is recovered from the observed splitting. This value is close to that found theoretically for the bulk viscosity of a liquid with solid inclusions (Stevenson, 1983), and recently, experimentally, for iron melts under high pressures by Brazhkin and Lyapin (2000). These results appear to confirm the semi-solid nature of the F-layer, long held to be the seat of the compositional convection driving the geodynamo. In addition, the mode periods are extremely sensitive to inner core density, increasing by 200 $minutes/gm \cdot cm^{-3}$. The recovered viscosity reproduces the observed equatorial mode periods to 6.48 s suggesting a density resolution of 4 parts in10^5.

Acknowledgments

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References

[1] Aldridge, K.D., Crossley, D., Mansinha, L. and D. E. Smylie (1991): GGP: The Global Geodynamics Project, Cahiers du Centre Européen de Géodynamique et de Séismologie, **3**, 169-196.

[2] Brazhkin, V. and A. Lyapin (2000): Universal Viscosity Growth in Metallic Melts at Megabar Pressures: the Vitreous State of the Earth's Inner Core, Physics-Uspekhi, **43**, 493-508.

[3] Bryan, G. H. (1889): The Waves on a Rotating Liquid Spheroid of Finite Ellipticity, Philos. Trans. R. Soc. London, Ser. A, **180**, 187-269.

[4] Bullen, K. E. and B. A. Bolt (1985): An Introduction to the Theory of Seismology, 4th edn., Cambridge University Press, 499p.

[5] Courtier, N., Ducarme, B., Goodkind, J., Hinderer, J., Imanishi, Y., Seama, N., Sun, H., Merriam, J., Bengert, B. and D. E. Smylie (2000): Global Superconducting Gravimeter Observations and the Search for the Translational Modes of the Inner Core, Phys. Earth Planet. Inter., **117**, 3-20.

[6] Dziewonski, A. and D. Anderson (1981): Preliminary Reference Earth Model, Phys. Earth Planet. Inter., **25**, 297-356.

[7] Gilbert, F. and A. Dziewonski (1975): An Application of Normal Mode Theory to the Retrieval of Structural Parameters and Source Mechanisms from Seismic Spectra, Philos. Trans. R. Soc. London, Ser. A, **278**, 187-269.

[8] Greenspan, H. P. (1969): The Theory of Rotating Fluids, Cambridge University Press, 328p.

[9] Moore, D. W. (1978): Homogeneous Fluids in Rotation, In: Roberts, P. H. and A. M. Soward (Eds.), Rotating Fluids in Geophysics, Academic Press, 551p.

[10] Smylie, D. E. and D. G. McMillan (2000): The Inner Core as a Dynamic Viscometer, Phys. Earth Planet. Inter., **117**, 71-79.

[11] Smylie, D. E. and D. G. McMillan (1998): Viscous and Rotational Splitting of the Translational Oscillations of Earth's Solid Inner Core, Phys. Earth Planet. Inter., **106**, 1-18.

[12] Smylie, D. E., Hinderer, J., Richter, B. and B. Ducarme (1993): The Product Spectra of Gravity and Barometric Pressure in Europe, Phys. Earth Planet. Inter., **80**, 135-157.

[13] Smylie, D. E.,Jiang, Xianhua, Brennan, B. J. and Kachishige Sato (1992): Numerical Calculation of Modes of Oscillation of the Earth's Core, Geophys. J. Int., **108**, 465-490. Smylie, D. E. and L. Mansinha (1971): The Elasticity Theory of Dislocations in Real Earth Models and Changes in the Rotation od the Earth, Geophys. J. R. astr. Soc., **23**, 329-354.

[14] Stevenson, D. J. (1983): Anomalous Bulk Viscosity of Two-Phase Fluids and Implications for Planetary Interiors, J. Geophys. Res., **88**, 2445-2455.

[15] Widmer, R., Masters, G. and F. Gilbert (1988): The Spherical Earth Revisited, 17th International Conference on Mathematical Geophysics, Blanes, Spain.